# 3D Line Mapping Revisited 

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#### Abstract

In contrast to sparse keypoints, a handful of line segments can concisely encode the high-level scene layout, as they often delineate the main structural elements. In addition to offering strong geometric cues, they are also omnipresent in urban landscapes and indoor scenes. Despite their apparent advantages, current line-based reconstruction methods are far behind their point-based counterparts. In this paper we aim to close the gap by introducing LIMAP, a library for 3D line mapping that robustly and efficiently creates 3D line maps from multi-view imagery. This is achieved through revisiting the degeneracy problem of line triangulation, carefully crafted scoring and track building, and exploiting structural priors such as line coincidence, parallelism, and orthogonality. Our code integrates seamlessly with existing point-based Structure-from-Motion methods and can leverage their 3D points to further improve the line reconstruction. Furthermore, as a byproduct, the method is able to recover 3D association graphs between lines and points / vanishing points (VPs). In thorough experiments, we show that LIMAP significantly outperforms existing approaches for $3 D$ line mapping. Our robust $3 D$ line maps also open up new research directions. We show two example applications: visual localization and bundle adjustment, where integrating lines alongside points yields the best results. Code is available at https://github.com/cvg/limap.


## 1. Introduction

The ability to estimate 3D geometry and build sparse maps via Structure-from-Motion (SfM) has become ubiquitous in 3D computer vision. These frameworks enable important tasks such as building maps for localization [60], providing initial estimates for dense reconstruction and refinement [65], and novel view synthesis [45, 48]. Currently, the field is dominated by point-based methods in which 2D keypoints are detected, matched, and triangulated into 3D maps [20,64]. These sparse maps offer a compact scene representation, only reconstructing the most distinctive points.

While there have been tremendous progress in pointbased reconstruction methods, they still struggle in scenes


Figure 1. In this paper, we propose a robust pipeline for mapping 3D lines (b), which offers stronger geometric clues about the scene layout compared to the widely used point mapping (a). Part of the success of our pipeline attributes to the modeling of structural priors such as coincidence (c), and parallelism / orthogonality (d). The corresponding 3D association graphs between lines and points / vanishing points (VPs) are also recovered from our system as a byproduct. The degree-1 point and degree- 2 junctions are colored in blue and red respectively in (c), while parallel lines associated with the same VP are colored the same in (d).
where it is difficult to detect and match sufficiently many stable keypoints, such as in indoor areas. On the contrary, these man-made scenes contain abundant lines, e.g. in walls, windows, doors, or ceilings. Furthermore, lines exhibit higher localization accuracy with less uncertainty in pixels [16]. Last but not least, lines appear in highly structured patterns, often satisfying scene-wide geometric constraints such as co-planarity, coincidence (line intersections), parallelism, and orthogonality. In practice, lines suffer from different issues, such as poor endpoint localization and partial occlusion. However, recent line detectors and matchers are bridging the gap of performance between points and lines [25, 46, 84], making it timely to revisit the line reconstruction problem.

Despite their rich geometric properties and abundance in the real world, there exist very few line-based reconstruction methods in the literature [22, 23, 44, 77]. In practical applications, they have also not achieved the same level of success as their point-based counterparts. We believe this is due to
several intrinsic challenges specific to line mapping:

- Inconsistent endpoints. Due to partial occlusion, lines often have inconsistent endpoints across images.
- Line fragmentation. In each image there might be multiple line segments that belong to the same line in 3D. This makes the process of creating track associations more complex compared to building 3D point tracks.
- No two-view geometric verification. While point matches can be verified in two views via epipolar geometry, lines require at least three views to filter.
- Degenerate configurations. In practice line triangulation is more prone to unstable configurations (see Fig. 8), e.g. becoming degenerate whenever the line is parallel with the camera motion (i.e. to epipolar lines).
- Weaker descriptor-based matching. State-of-the-art descriptors for line segments are far behind their point-based counterparts, putting more emphasis on geometric verification and filtering during reconstruction.
In this paper we aim to reduce the gap between pointbased and line-based mapping solutions. We propose a new robust mapping method, LIMAP, that integrates seamlessly into existing open-source point-based SfM frameworks [64, 67,80 ]. By sharing the code with the research community we hope to enable more research related to lines; both for low-level tasks (such as improving line segment detection and description) and for integrating lines into higher-level tasks (such as visual localization or dense reconstruction). In particular, we make the following contributions in the paper:
- We build a new line mapping system that reliably reconstructs 3D line segments from multi-view RGB images. Compared to previous approaches, our line maps are significantly more complete and accurate, while having more robust 2D-3D track associations.
- We achieve this by automatically identifying and exploiting structural priors such as coincidence (junctions) and parallelism. Our technical contribution spans all stages of line mapping including triangulating proposals, scoring, track building, and joint optimization, with 3D line-point / VP association graphs output as a byproduct.
- The framework is flexible such that researchers can easily change components (e.g. detectors, matchers, vanishing point estimators, etc.) or integrate additional sensor data (e.g. depth maps or other 3D information).
- We are the first to go beyond small test sets by quantitatively evaluating on both synthetic and real datasets to benchmark the performance, with hundreds of images for each scene, in which LIMAP consistently and significantly outperforms existing approaches.
- Finally, we demonstrate the usefulness of having robust line maps by showing improvement over purely pointbased methods in tasks such as visual localization and bundle adjustment in Structure-from-Motion.


## 2. Related Work

Line Detection and Matching. Detecting 2D line segments conventionally relies on grouping image gradients [5, 75]. To improve the robustness and repeatability, learning-based line detectors were later proposed to tackle the problem of wireframe parsing [ $25,43,82,83,88,90$ ]. Recent deep detectors [26,46,81] manage to achieve impressive results for detecting general line segments. Matching of the detected line segments is often based on comparing either handcrafted [ $8,74,76,85]$ or learning-based $[1,34,46,73,84]$ descriptors. Some recent methods also exploit point-line [14, 15] and line-junction-line structures $[38,39]$ to improve matching results, yet still not reaching the reliability level of advanced point matchers [58, 70]. Our method can leverage any line detector and matcher, and is robust to outliers.

Line Reconstruction. As a seminal work, Bartoli and Sturm [6, 7] proposed a full SfM pipeline for line segments, later improved by Schindler [63] with Manhattan-world assumption [12]. Jain et al. [27] proposed to impose global topological constraints between neighboring lines, which were further explored in $[51,53,54]$ to build wireframe models. Some learning-based methods [42,90] were introduced as well to predict 3D wireframes. Hofer et al. [21-23] proposed checking weak epipolar constraints over exhaustive matches and graph clustering, and introduced the Line3D++ software (referred as $\mathrm{L} 3 \mathrm{D}++$ in this paper), which remains the top choice $[17,42]$ for acquiring 3D line maps so far. Recently, ELSR [77] employed planes and points to guide the matching. However, all prior work mainly shows qualitative results and provides quantitative evaluation only on relatively small image sets [27,69]. In this paper, we set up a quantitative evaluation on benchmarks with hundreds of images, where our proposed system significantly surpasses prior work by improving all stages in the mapping pipeline.

Line-based Applications. The resulting 3D line maps can be used for many downstream applications. [23] advocates the complementary nature of line reconstruction for structure visualization. Some incremental line-based SfM systems are introduced in [24,44, 86]. To improve quality and robustness, recent methods [18,19,40,41,49,78,91] jointly employ point and line features in SLAM. While their line maps are often noisy and incomplete, noticeable improvement has been achieved in the accuracy of the recovered camera motion. There has also been development on VP estimation [9,37,50, 87] and solvers for joint point-line pose estimation [4,52, 72, 89]. Recently, promising performance in visual localization has been achieved by combining point and line features in a refinement step [17]. In this paper, we show that our line maps can benefit multiple applications such as localization, SfM, and MVS (Sec. J in supp.). In particular, we present very competitive results on point-line visual localization.


Figure 2. Overview. Given a set of posed images and optional 3D points, we associate nearby points to lines, match the lines, triangulate them with 4 different strategies, score 3D line proposals, build line tracks, jointly optimize all features, before obtaining our final reconstruction.

## 3. The Proposed 3D Line Mapping Pipeline

We now present our proposed pipeline for 3D line mapping. Our method takes as input a set of images with 2D line segments from any existing line detectors. We assume the camera pose for each image is available (e.g. from SfM/SLAM), and optionally we can also leverage a 3D point cloud (e.g. obtained from point-based SfM). The pipeline consists of three main steps:

- Proposal Generation (Sec. 3.1): For each 2D line segment, we generate a set of 3D line segment proposals.
- Scoring and Track Association (Sec. 3.2): Considering multi-view consistency, we score each proposal, select the best candidate for each 2D line, and associate them into a set of 3D line tracks.
- Joint Refinement (Sec. 3.3): Finally, we jointly perform non-linear refinement over the 3D line tracks along with 3D points and VP directions, integrating additional structural priors as soft constraints.

Figure 2 shows an overview of the overall pipeline. In the following sections, we detail each of the three main steps.

By design our pipeline is robust to scale changes and we use the same hyper-parameters for all experiments across datasets, which are provided in Sec. F. 2 in the supp.

### 3.1. Generating 3D Line Segment Proposals

The first step is to generate a set of 3D line proposals for each 2D line segment. Given a segment in an image, we use any existing line matcher to retrieve the top $K$ line matches in each of the $n_{v}$ closest images. Using the top $K$ line matches instead of a single match increases the chance of getting a correct match, while wrong matches will be filtered out in subsequent steps.

Let $\left(\boldsymbol{x}_{1}^{r}, \boldsymbol{x}_{2}^{r}\right) \in \mathbb{R}^{3} \times \mathbb{R}^{3}$ be the two endpoints (in homogeneous coordinates normalized by the intrinsics) for the reference line segment that we wish to generate proposals for. For ease of notation, we let the world-coordinate system align with the reference view. The endpoints of the 3D line proposals that we generate can all be written as

$$
\begin{equation*}
\boldsymbol{X}_{1}=\lambda_{1} \boldsymbol{x}_{1}^{r}, \quad \boldsymbol{X}_{2}=\lambda_{2} \boldsymbol{x}_{2}^{r} \tag{1}
\end{equation*}
$$

for some values of $\lambda_{1}, \lambda_{2} \in \mathbb{R}$. Having the 3D endpoints of all proposals lie on the camera rays of the 2D endpoints simplifies the scoring procedure in the second step (Sec. 3.2).

### 3.1.1 Line Triangulation

For each matched 2D line segment $\left(\boldsymbol{x}_{1}^{m}, \boldsymbol{x}_{2}^{m}\right)$ we generate one proposal via algebraic line triangulation. Let $\left(R^{m}, \boldsymbol{t}^{m}\right)$ be the camera pose of the matched view. We can then solve linearly for the endpoint ray depths $\lambda_{i}$ as

$$
\begin{equation*}
\left(\boldsymbol{x}_{1}^{m} \times \boldsymbol{x}_{2}^{m}\right)^{T}\left(R^{m}\left(\lambda_{i} \boldsymbol{x}_{i}^{r}\right)+\boldsymbol{t}^{m}\right)=0, \quad i=1,2 . \tag{2}
\end{equation*}
$$

The proposals are then filtered with cheirality checks (positive $\lambda$ ) and degeneracy check via the angle between ray $\boldsymbol{x}_{i}^{r}$ and $\boldsymbol{\ell}_{\boldsymbol{m}}=\boldsymbol{x}_{1}^{m} \times \boldsymbol{x}_{2}^{m}$. Note that line triangulation becomes inherently unstable close to degenerate configurations when $\ell_{m}^{T} R^{m} \boldsymbol{x}_{i}^{r}=0$, where we get zero or infinite solutions from (2). Geometrically, this happens when the line is parallel with the epipolar plane: If $\boldsymbol{\ell}_{m}^{T} \boldsymbol{t}^{m} \neq 0$ they have no intersection, otherwise they intersect fully and we get infinite solutions $\boldsymbol{\ell}_{m} \sim \boldsymbol{t}^{m} \times R^{m} \boldsymbol{x}_{i}^{r}=E \boldsymbol{x}_{i}^{r}$, i.e. the line segment coincides with the epipolar line from $\boldsymbol{x}_{i}^{r}$. This issue is further illustrated in Figure 8. Since we solve for each $\lambda_{i}$ independently, the triangulation problem can have zero, one, or two degenerate endpoints. We term the case with one degenerate endpoint as a weakly degenerate one, and the case with two degenerate endpoints as fully degenerate. In contrast to the point case, two-view line triangulation is minimal such that any solution fits the measurements exactly with zero error, preventing filtering with 2 D reprojection error at this stage.

### 3.1.2 Point-Line Association

To obtain meaningful proposals in degenerate cases, we leverage additional geometric information coming from either points or associated vanishing points (VPs). 2D-3D point correspondences can either come from a point-based SfM model or be triangulated from matched endpoints/junctions. For each 2D line segment, we associate all 2D points within a fixed pixel threshold and thereby associate with their corresponding 3D points. For each image, we also estimate a set of VPs and their association to 2D lines using JLinkage [71].

### 3.1.3 Point-guided Line Triangulation

We now generate a second set of proposals for each 2D line segment with the assistance of the associated 2D-3D point correspondences and vanishing points. In the following parts we present three different methods. M1 employs multiple associated 3D points so it is stable for all cases including the fully degenerate ones, while M2 and M3 with one known point / VP can help generate stable proposals in weakly degenerate cases, which are more common in practice. Cheirality tests are applied to all proposals with respect to both views.
M1. Multiple Points. For each matched line segment we generate one proposal by collecting all of the associated 3D points that are common between the reference and the match. On top of those common points, we fit a 3D line that is then projected onto two camera rays corresponding to $\boldsymbol{x}_{1}^{r}$ and $\boldsymbol{x}_{2}^{r}$. M2. Line + Point. For each matched line segment we also generate one proposal for each shared 3D point. We first project the 3D point onto the plane spanned by $\boldsymbol{x}_{1}^{r}$ and $\boldsymbol{x}_{2}^{r}$. We then aim to find a line that passes through the projection and minimizes the residuals in (2) to the matched line. This can be formulated as a quadratic optimization problem in the two endpoint depths $\boldsymbol{\lambda}=\left(\lambda_{1}, \lambda_{2}\right)$ with a single constraint:

$$
\begin{equation*}
\min _{\boldsymbol{\lambda} \in \mathbb{R}^{2}} \boldsymbol{\lambda}^{T} A \boldsymbol{\lambda}+\boldsymbol{b}^{T} \boldsymbol{\lambda}, \quad \text { s.t. } \quad \boldsymbol{\lambda}^{T} Q \boldsymbol{\lambda}+\boldsymbol{q}^{T} \boldsymbol{\lambda}=0 . \tag{3}
\end{equation*}
$$

Due to the low-dimensionality of the problem, a closed-form solution can be derived by reducing it to a univariate quartic polynomial. We show the full derivation in Sec. B in supp. M3. Line + VP. Each VP corresponds to a 3D direction. For each associated VP, we generate one proposal based on its direction (again projected onto the plane spanned by $\boldsymbol{x}_{1}^{r}$ and $\boldsymbol{x}_{2}^{r}$ ). This gives a single linear constraint on the ray depths,

$$
\begin{equation*}
\left(\boldsymbol{v} \times\left(\boldsymbol{x}_{1}^{r} \times \boldsymbol{x}_{2}^{r}\right)\right)^{T}\left(\lambda_{2} \boldsymbol{x}_{2}^{r}-\lambda_{1} \boldsymbol{x}_{1}^{r}\right)=0 . \tag{4}
\end{equation*}
$$

where $\boldsymbol{v} \in \mathbb{R}^{3}$ is the VP. Using the constraint, we then solve for $\boldsymbol{\lambda}=\left(\lambda_{1}, \lambda_{2}\right)$ by minimizing the two residuals of (2) in a least squares sense. Note that $\boldsymbol{v}$ can either come from the reference image, or from a matched line in another image. Extension: Line Mapping Given Depth Maps. The proposal generation step can be improved when each image has a corresponding depth map (e.g. from an RGB-D sensor), which can be leveraged with robust line fitting to generate the 3D line proposals. Refer to Sec. E in our supplementary material for more details and results.

### 3.2. Proposal Scoring and Track Association

At this point, each 2D line segment $l$ in image $I$ is associated with a set $\mathcal{K}$ of 3 D line segment proposals (stemming from the top $K$ line matches and various triangulations) for each neighboring image $J$. We describe in the following how we select the best 3D line proposal for each 2D line segment, and associate these lines into tracks. For each of these


Figure 3. Scoring methods. We propose three novel line scoring measures that are scale-invariant and handle different line lengths. steps, we leverage different scoring methods quantifying the distance between two 3D line segments $\left(L_{1}, L_{2}\right)$. These distances are usually computed symmetrically and averaged, and can be obtained both in 3D and in 2D by projecting each 3D line into the other view. We start by presenting two classic ones, and then define our three novel line distances (one for 3D proposal selection and two for track building).

- Angular distance: angle between $L_{1}$ and $L_{2}$.
- Perpendicular distance: maximum orthogonal distance of the endpoints of $L_{1}$ to the infinite line spanned by $L_{2}$.

3D Proposal Selection. To select best 3D candidate for each 2D line, we score each proposal $L_{i}$ by measuring its consistency with the others. Here we introduce a new distance:

- Perspective distance: assuming the endpoints of $L_{1}$ and $L_{2}$ are on the same rays as in Fig. 3(a), the distance is defined as the endpoint distances, divided by the ray depths $d_{s}, d_{e}$ of the endpoints of $L_{1}$ in image 1 . This score can filter out ill-posed triangulations (refer to Sec. F. 3 in supp. for detailed discussions), while remaining scale-invariant.

This new distance, together with the angular distance in 2D and 3D, and the perpendicular distance in 2D, have different scales. In order to aggregate them together, we associate a scaling factor $\tau_{r}$ to each distance $r$ and get a normalized score $s_{n}=e^{-\left(r / \tau_{r}\right)^{2}} \in(0,1]$. Denoting by $\mathcal{S}$ the set of all the corresponding normalized scores and $\mathbb{1}$ the indicator function, the score between $L_{1}$ and $L_{2}$ becomes

$$
\begin{equation*}
s\left(L_{1}, L_{2}\right)=\min _{s_{n} \in \mathcal{S}}\left(s_{n} \cdot \mathbb{1}_{s_{n} \geq 0.5}\right) \in\{0\} \cup[0.5,1] . \tag{5}
\end{equation*}
$$

Now equipped with unique score per line pair, we can consider all the neighboring 3D line candidates $L_{j}^{k}$ coming from the neighboring image $J$ and proposal $k$. The consistency score is defined by summing the best score from each image:

$$
\begin{equation*}
s_{c}\left(L_{i}\right)=\sum_{J \in \mathcal{N}_{I}} \max _{k \in \mathcal{K}} s\left(L_{i}, L_{J}^{k}\right) \tag{6}
\end{equation*}
$$

where $\mathcal{N}_{I}$ is the set of neighboring images of $I$. The best 3 D line candidate for each 2D line segment $l$ is then selected as the proposal with the highest score: $L=\operatorname{argmax}_{L_{i}} s_{c}\left(L_{i}\right)$. If the score is less than 1.0, i.e. the best candidate has less than two supports from neighboring views, we ignore this 2 D line segment in the subsequent track building process.

Track Building. At this point, each 2D segment has been assigned a unique 3D line (its best 3D line candidate). The goal of this step is to gather these 2D segments into line tracks. For this, we form a graph where the 2D segments are nodes and all initial line matches are edges. We aim to prune edges in the graph such that the connected 2D segments share similar 3D assignments. We propose two new line scoring measures that can cope with different endpoint configurations and variable scales across images.

- Overlap score: we project $L_{1}$ orthogonally onto $L_{2}$, clip the projected endpoints to the endpoints of $L_{2}$ if they fall outside of $L_{2}$ to get segment $\Pi\left(L_{1}\right)$, and compare the ratio of lengths to a threshold $\tau_{o}: \mathbb{1}_{\frac{\left|\Pi\left(L_{1}\right)\right|}{\left|L_{2}\right|} \geq \tau_{o}}$ (see Fig. 3(b)).
- InnerSeg distance: the endpoints of $L_{1}$ are perpendicularly unprojected to $L_{2}$. If they fall outside of $L_{2}$, we clip them to the closest endpoint of $L_{2}$. By doing this in both directions, we can define two inner segments (see Fig. 3(c)), and the InnerSeg distance as the maximum distance between their endpoints. To make this measure scale-invariant, we additionally divide it by a scale factor $\sigma=\frac{\min \left(d_{1}, d_{2}\right)}{f}$, where $d_{j}$ is the depth of the mid-point of $L_{j}$ in image $J$ and $f$ is the focal length. This encodes how far the mid-point can move in 3D before reaching 1 pixel error in the image (detailed in Sec. F. 3 in supp.).

We then convert the InnerSeg distance computed in 3D to a normalized score as in the previous paragraph, and combine it with the overlap score in 2D and 3D and previous scores using (5). Given these pairwise scores of 3D lines, we can now prune edges whose score is below a threshold $t_{f}=0.5$. The connected components of the resulting graph yield the line tracks, ignoring components with less than 3 nodes.

For each track, we then re-estimate a single 3D line segment. Using the set of endpoints from the 3D assignments of all nodes in the track, we apply Principal Component Analysis (PCA) and use the principal eigenvector and mean 3D point to estimate the infinite 3D line. We then project all endpoints on this infinite line to get the new 3D endpoints.

### 3.3. Joint Optimization of Lines and Structures

Finally, we perform non-linear refinement on the acquired 3D lines with their track information. The straightforward approach is to perform geometric refinement on the reprojection error. With the 2D point-line association available, we can formulate a joint optimization problem by including additional structural information. The energy to minimize can be written as follows:

$$
\begin{equation*}
E=\sum_{p} E_{P}(p)+\sum_{l} E_{L}(l)+\sum_{(p, l)} E_{P L}(p, l) \tag{7}
\end{equation*}
$$

where $E_{P}$ and $E_{L}$ are the data terms, and $E_{P L}$ encodes the 3 D association between lines and points / VPs. In particular,
$E_{P}$ is the 2D point reprojection error as in regular bundle adjustment [64]. The association energy is softly weighted (as discussed later) and optimized with robust Huber loss [3]. Each line is converted into a 4-DoF infinite line with Plücker coordinate [7] for optimization and converted back to line segments by unprojecting its 2D supports. Each vanishing point is parameterized with a 3-dimensional homogeneous vector. Refer to Sec. A in supp. for details on efficient computation with minimal parameterization.
Geometric Refinement. The data term of each line track is also defined on its 2 D reprojections. In particular, we measure the 2D perpendicular distance weighted by the angle consistency, which we robustly equip with Cauchy loss [3]:

$$
\begin{equation*}
E_{L}(l)=\sum_{k} w_{\angle}^{2}\left(L_{k}, \ell_{k}\right) \cdot e_{\text {perp }}^{2}\left(L_{k}, \ell_{k}\right) \tag{8}
\end{equation*}
$$

where $e_{\text {perp }}$ is the perpendicular distance, $L_{k}$ is the 2 D projection of the 3D segment, $\ell_{k}$ are the 2 D line segments, and $w_{\angle}$ is the exponential of one minus the cosine of the 2 D angle between the projected and the observed line.
Soft Association between Lines and Points. For each pair of 3D line and 3D point with their track information, we can estimate how likely they are spatially associated by traversing the 2D association graph (described in Sec. 3.1.2) of their supports. Specifically, we count the number of associations among the 2D supports of the line track and point track, and keep pairs with at least three 2D associations. The 3D association energy $E_{P L}$, defined on the surviving pairs, is formulated as the 3D point-line distance weighted by the number of 2D associations on their supports.
Soft Association between Lines and VPs. Same as the point case, we can also build a soft association problem between lines and VPs. First, we acquire 3D VP tracks by transitively propagating line correspondences from the 3D line tracks. Then, we count the number of associations among the 2D supports for each pair of 3D line and VP track. The 3D line-VP association energy is defined as the sine of the direction angle between the 3D line and the VP, implicitly enforcing parallelism. Furthermore, we add regularizations to the nearly orthogonal VP pairs to enforce orthogonality of different line groups. Refer to Sec. C in supp. for details.

## 4. Experiments

Implementation Details. Our whole library is implemented in C++ with Python bindings [28]. The triangulation and scoring can be run in parallel for each node, enabling scalability to large datasets. We use $n_{v}=20$ visual neighbors and keep the top $K=10$ line matches. We provide all the values of thresholds and scaling factors in Sec. F. 2 in supp.

### 4.1. Line Mapping

To validate the effectiveness of our system, we set up an evaluation benchmark to quantify the quality of the recon-

| Line type | Method | R1 | R5 | R10 | P1 | P5 | P10 | \# supports |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LSD | L3D++ [23] | 37.0 | 153.1 | 218.8 | 53.1 | 80.8 | $\mathbf{9 0 . 6}$ | $(14.8 / 16.8)$ |
|  | ELSR [77] | 13.9 | 59.7 | 96.5 | 55.4 | 72.6 | 82.2 | (N/A / N/A) |
|  | Ours | $\mathbf{4 8 . 6}$ | $\mathbf{1 8 5 . 2}$ | $\mathbf{2 5 1 . 3}$ | $\mathbf{6 0 . 1}$ | $\mathbf{8 2 . 4}$ | 90.0 | $(\mathbf{1 6 . 4} / \mathbf{2 0 . 5})$ |
| SOLD2 | L3D++ [23] | 36.9 | 107.5 | 132.8 | 67.2 | $\mathbf{8 6 . 8}$ | $\mathbf{9 3 . 2}$ | (13.2 / 20.4) |
| [46] | Ours | $\mathbf{5 4 . 3}$ | $\mathbf{1 5 1 . 1}$ | $\mathbf{1 9 1 . 2}$ | $\mathbf{6 9 . 8}$ | 84.6 | 90.0 | $(\mathbf{1 6 . 5}$ / 38.7) |

Table 1. Line reconstruction on Hypersim [55] with LSD [75] and SOLD2 [46] lines. $R \tau$ and $P \tau$ are reported at $1 \mathrm{~mm}, 5 \mathrm{~mm}, 10$ mm along with the average number of supporting images/lines.

| Method | R5 | R10 | R50 | P5 | P10 | P50 | \# supports |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| L3D++ [23] | 373.7 | 831.6 | 2783.6 | 40.6 | 54.5 | 85.9 | (8.8 / 9.3) |
| ELSR [77] | 139.2 | 322.5 | 1308.0 | 38.5 | 48.0 | 74.5 | (N/A / N/A) |
| Ours (line-only) | 472.1 | 1058.8 | 3720.7 | $\mathbf{4 6 . 8}$ | $\mathbf{5 8 . 4}$ | $\mathbf{8 6 . 1}$ | (10.3 / 11.8) |
| Ours | $\mathbf{5 0 8 . 3}$ | $\mathbf{1 1 5 4 . 5}$ | $\mathbf{4 1 7 9 . 5}$ | 46.0 | 56.9 | 83.7 | (10.4 / 12.0) |

Table 2. Line reconstruction on train split of Tanks and Temples [32] with LSD [75] lines. $R \tau$ and $P \tau$ are reported at $5 \mathrm{~mm}, 10 \mathrm{~mm}$, 50 mm along with the average number of supporting images/lines.
structed 3D line maps. As there are no ground truth (GT) 3D lines, we evaluate the 3D line mapping with either GT mesh models or point clouds. We use the following metrics: - Length recall (in meters) at $\tau(R \tau)$ : sum of the lengths of the line portions within $\tau \mathrm{mm}$ from the GT model.

- Inlier percentage at $\tau(P \tau)$ : the percentage of tracks that are within $\tau \mathrm{mm}$ from the GT model.
- Average supports: average number of image supports and 2D line supports across all line tracks.
In the following, we compare our system with two state-of-the-art methods as baselines: L3D++ [23] and ELSR [77], using two line detectors: the traditional LSD detector [75] and the learning-based SOLD2 [46]. For ELSR [77], we convert the input into VisualSfM [80] format and use code ${ }^{1}$ from the authors (only supporting LSD [75]).

Our first evaluation is run on the first eight scenes of the Hypersim dataset [55], composed of 100 images each, and is reported in Tab. 1. For both detectors, we reconstruct much more complete line maps with better or comparable precision than the competitors, while also exhibiting significantly higher quality of track information. This abundant track association is beneficial particularly for line-based applications such as visual localization [17]. After discussing with the authors of ELSR, it seems that their method does not achieve satisfactory results due to a lack of point and plane features.

We further evaluate all three methods on the train split of the Tanks and Temples dataset [32] without Ignatius as it has no line structures. As SOLD2 [46] is trained for indoor images, we only use LSD [75]. Since the provided point cloud was cleaned to focus only on the main subject, we compute its bounding box, extend it by one meter, and only evaluate lines inside this region. This prevents incorrectly penalizing correct lines that are far away from the main scene,

[^0]

Figure 4. Top row: L3D++ [23]. Bottom row: Ours. Both systems are run on Horse and Family from [32]. We show two different views on the main scene of Horse.


Figure 5. Qualitative results on Hypersim [55] and Tanks and Temples [32]. On Barn we jointly visualize our results and the aligned ground truth point cloud.


Figure 6. Qualitative results of the recovered line-point and lineVP association graphs (visualized similarly as in Fig. 1).
which our method is particularly good at thanks to our scaleinvariant design (refer to Sec. G in supp.). Tab. 2 shows the results, where our methods significantly improve the mapping quality across the board. Fig. 4 shows qualitative comparison between our method and L3D++ [23]. Our results exhibit better completeness, have less noisy lines that are flying around, and achieve significantly more robust reconstructions of subtle details (e.g. on the ground). More examples of our produced line maps are shown in Fig. 5.

As an additional output of our system, junction structures and line-line relations such as parallelism and orthogonality are discovered, as shown in Fig. 6. This directly comes from the line-point and line-VP soft associations of Sec. 3.3. From the recovered structures, we can clearly perceive the scene and easily recognize the main Manhattan directions [12].

To demonstrate the scalability of the proposed system, we also run our method on two large-scale datasets: Aachen (6,697 images) [61, 62] and Rome city (16,179 images) [2, $67,68]$. Fig. 7 shows that our method produces reliable line maps with clear structures. Note that the camera poses from Bundler [67] on Rome city are far from perfect, while our mapping still works reasonably well. The efficiency


Figure 7. Scalability to large-scale datasets: Aachen (6,697 images) [61] and Rome (16,179 images) [2, 67, 68]. For Aachen [61], parallel lines from the line-VP association graph are colored the same. For Rome [2,67,68], we visualize 10 representative components individually.


Figure 8. Uncertainty in line triangulation measured by the largest eigenvalue of the covariance (Sec. D in supp.). Left: Each segment is colored by the uncertainty in the triangulation. Lines that align with the epipolar lines (shown in blue) exhibit higher (red) uncertainty. Right: We perform a small synthetic experiment to illustrate this. The graph shows the uncertainty for line triangulation as the lines approach the degenerate state. We compare with pointbased triangulation assuming that endpoints are consistent.

| Line type | Triangulation | R1 | R5 | R10 | P1 | P5 | P10 | \# supports |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LSD | Endpoints | 27.6 | 101.4 | 138.0 | 58.2 | $\mathbf{8 3 . 5}$ | $\mathbf{9 2 . 1}$ | $(13.0 / 13.2)$ |
| [75] | Line | $\mathbf{4 8 . 3}$ | $\mathbf{1 8 7 . 0}$ | $\mathbf{2 5 7 . 4}$ | $\mathbf{5 9 . 2}$ | 81.9 | 89.8 | $(\mathbf{1 5 . 8} / \mathbf{1 9 . 1})$ |
| SOLD2 | Endpoints | 27.3 | 82.8 | 106.5 | 68.2 | 84.5 | 90.9 | $(12.3 / 19.9)$ |
| [46] | Line | $\mathbf{5 0 . 8}$ | $\mathbf{1 4 3 . 5}$ | $\mathbf{1 8 0 . 8}$ | $\mathbf{7 4 . 4}$ | $\mathbf{8 6 . 9}$ | $\mathbf{9 1 . 2}$ | $(\mathbf{1 5 . 1} / \mathbf{3 2 . 2})$ |

Table 3. Comparison between endpoint and line triangulation on Hypersim [55]. While being more stable at triangulation, the endpoints are often unmatched between line pairs.

| Line | M1 | M2 | M3 | R1 | R5 | R10 | P1 | P5 | P10 | \# supports |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\checkmark$ |  |  |  | 50.8 | 143.5 | 180.8 | 74.4 | 86.9 | 91.2 | $(15.1 / 32.2)$ |
|  | $\checkmark$ |  |  | 24.9 | 72.5 | 95.8 | 65.9 | 81.2 | 88.5 | $(11.3 / 15.7)$ |
|  | $\checkmark$ | $\checkmark$ |  | 37.7 | 116.8 | 152.6 | 71.0 | 84.2 | 89.7 | $(13.8 / 25.8)$ |
| $\checkmark$ | $\checkmark$ |  |  | 51.5 | 146.9 | 185.4 | 71.7 | 85.4 | 90.1 | $(14.9 / 31.2)$ |
| $\checkmark$ | $\checkmark$ | $\checkmark$ |  | 51.3 | 146.4 | 186.4 | 73.4 | 85.7 | 90.5 | $(15.8 / 35.6)$ |
| $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | 51.4 | 145.4 | 184.9 | 74.1 | 86.1 | 90.6 | $(16.5 / 38.7)$ |

Table 4. Ablation study on different types of triangulation proposals (defined in Sec. 3.1.3) on Hypersim [55] with SOLD2 [46].
bottleneck is in line detection and matching (we use SOLD2 [46] descriptors), while the rest of the mapping takes only $\sim 10$ minutes on Aachen [61,62]. The time complexity of our system is nearly linear with the number of images.

### 4.2. More Insights and Ablation Studies

Line Triangulation. To study the stability of the triangulation, we perform a small test on a stereo pair from AdelaideRMF [79] on the uncertainty (measured by the largest singular value of the covariance) of the triangulated 3D segments. We further run a synthetic experiment by generating random lines on a plane orthogonal to the stereo pair, and plot the uncertainty of point and line triangulations with

| Line type | Method | R1 | R5 | P1 | P5 | \# supports |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { LSD } \\ & {[75]} \end{aligned}$ | L3D++ [23] | 37.0 | 153.1 | 53.1 | 80.8 | (14.8 / 16.8) |
|  | Ours (line) w/ [23] scoring | 48.6 | 186.0 | 56.5 | 80.6 | (14.4/16.8) |
|  | Ours (line) w/ [23] merging | 41.2 | 158.2 | 59.6 | 82.5 | (15.6 / 16.7) |
|  | Ours (line) w/ exhaustive | 46.7 | 177.2 | 57.6 | 80.9 | (16.8 / 20.8) |
|  | Ours (line) | 48.3 | 187.0 | 59.2 | 81.9 | (15.8/19.1) |
| $\begin{gathered} \text { SOLD2 } \\ \hline \text { [46] } \end{gathered}$ | L3D++ [23] | 36.9 | 107.5 | 67.2 | 86.8 | (13.2 / 20.4) |
|  | Ours (line) w/ [23] scoring | 45.8 | 133.2 | 72.6 | 85.9 | (15.0 / 31.1) |
|  | Ours (line) w/ [23] merging | 37.7 | 113.4 | 70.5 | 84.5 | (13.3/23.9) |
|  | Ours (line) w/ exhaustive | 48.9 | 139.7 | 72.9 | 85.7 | (16.2 / 36.9) |
|  | Ours (line) | 50.8 | 143.5 | 74.4 | 86.9 | (15.1 / 32.2) |

Table 5. Studies on different components of our method with only line-line proposals against L3D++ [23].
respect to the angle of the lines with the baseline (refer to Sec. D in supp. for details). The results in Fig. 8 show that when the matched line is nearly parallel to the epipolar line, the line triangulation becomes degenerate with exploding uncertainty, while triangulating the endpoints is significantly more stable. Thus, combining points and VPs from the 2D association is beneficial to improve the stability of the proposals. However, the endpoints are generally not consistent across line matches in practice and need to be complemented with line-line triangulation. This can be verified in Tab. 3 where the performance significantly drops when we change line triangulation into endpoint triangulation.

We further ablate our four types of triangulation for generating proposals. Results in Tab. 4 show that integrating points and VPs enhance the 3D line maps, in particular significantly improving the track quality. Another surprising fact is that the third line in the table, relying only on points and line + point triangulation, already achieves better results than the prior baselines in Tab. 1. Employing all four types of proposals obtains the best trade-off.
Scoring and Track Building. We first study the effects of using exhaustive line matching as in L3D++ [23]. To enable direct comparison we only use line triangulation proposals. Results are shown in Tab. 5. While there are more proposals generated from the exhaustive matches, both the recall and precision decrease by a noticeable margin. This is probably due to the large number of wrong proposals misleading the scoring process. Nevertheless, our method with exhaustive matches still works significantly better than L3D++ [23]. To further study the effects of the proposed distance measurements at scoring and track building (merging), we re-implement the ones proposed in L3D++ [23] and perform direct comparison. Both our scoring and track

| Method | R1 | R5 | R10 | P1 | P5 | P10 | \# supports |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Line-only w/o refine | 43.5 | 135.8 | 180.1 | 75.1 | 87.2 | 92.2 | $(15.1 / 32.2)$ |
| Line-only w/ geom alone | 50.8 | 143.5 | 180.8 | 74.4 | 86.9 | 91.2 | $(15.1 / 32.2)$ |
| w/o refine | 46.5 | 146.0 | 189.7 | $\mathbf{7 6 . 8}$ | $\mathbf{8 8 . 9}$ | $\mathbf{9 3 . 3}$ | $\mathbf{( 1 6 . 5 / \mathbf { 3 8 . 7 } )}$ |
| w/ geom alone | 51.4 | 145.4 | 184.9 | 74.1 | 86.1 | 90.6 | $\mathbf{( 1 6 . 5 / \mathbf { 3 8 . 7 } )}$ |
| w/ joint optimization | $\mathbf{5 4 . 3}$ | $\mathbf{1 5 1 . 1}$ | $\mathbf{1 9 1 . 2}$ | 69.8 | 84.6 | 90.0 | $\mathbf{( 1 6 . 5 / \mathbf { 3 8 . 7 } )}$ |

Table 6. Line refinement on Hypersim [55] with SOLD2 [46].

| Dataset | $\operatorname{HLoc}^{2}[56,57]$ | PtLine [17] | Ours |
| :--- | :---: | :---: | :---: |
| Cambridge [30] | $7.0 / 0.13 / 44.0$ | $7.4 / 0.13 / 43.5$ | $\mathbf{6 . 7 / 0 . 1 2 / 4 6 . 1}$ |
| 7Scenes [66] | $3.3 / 1.08 / 73.0$ | $3.3 / 1.09 / 72.7$ | $\mathbf{3 . 0} / \mathbf{1 . 0 0 / 7 8 . 0}$ |

Table 7. Visual localization on Cambridge [31] and 7Scenes [66]. We report the median translation and rotation errors in cm and degrees, and the pose accuracy (\%) at $5 \mathrm{~cm} / 5 \mathrm{deg}$ threshold. All metrics are averaged across all scenes of each dataset.


HLoc [56,57]

|  |  |  |
| :--- | :---: | :---: |
| HLoc [57] $/ R)$ err. $\downarrow$ | Acc. $\uparrow$ |  |
| HLoc [57] w/ depth | $5.2 / 1.46$ | 46.8 |
| PtLine [17] | 4.25 | 53.4 |
| Ours w/ L3D++ [23] | $4.1 / 1.33$ | 51.9 |
| Ours w/ LIMAP | $\mathbf{3 . 7} / \mathbf{1 . 0 2}$ | $\mathbf{7 1 . 1}$ |

Figure 9. Line-assisted Visual localization on Stairs from 7Scenes [66]. Blue: 2D points/lines; Green/Red: Projected 3D points/lines.
building are significantly better, especially when equipped with SOLD2 [46] which produces more structured lines.
Joint Optimization. Finally, we ablate the proposed joint optimization in our pipeline. First, we remove the point-line association and only apply the geometric residuals (reprojection error). Results in Tab. 6 show that the geometric refinement improves significantly when the proposals solely come from line triangulation. However, when adding additional proposals from points and VPs, it contributes marginally and even misleads some lines that are generated from points and VPs but poorly conditioned for lines (R10 decreases). When integrated with joint optimization with soft association, the recall is further improved noticeably, while sacrificing a bit on the precision. It is worth pointing out that the joint optimization also enables the byproduct of junction structures and line-line relations (e.g. in Fig. 6).

### 4.3. Applications

Line-Assisted Visual Localization. We build a hybrid visual localization with both points and lines on top of the acquired 3D line maps. Specifically, we first build point maps as in HLoc [56,57] and line maps with our proposed method. Then, we match points and lines respectively and get 2D-3D correspondences from the track information in the 3D maps. Given these correspondences, we combine

[^1]|  | Med. error $\downarrow$ | AUC @ $\left(1^{\circ} / 3^{\circ} / 5^{\circ}\right) \uparrow$ |
| :--- | :---: | :---: |
| COLMAP [64] | 0.188 | $77.3 / 89.0 / 91.6$ |
| COLMAP [64] + LIMAP refinement | $\mathbf{0 . 1 4 6}$ | $\mathbf{8 2 . 9 / 9 1 . 2 / 9 3 . 0}$ |

Table 8. Joint bundle adjustment of points and lines on Hypersim [55]. Relative pose errors are measured on all image pairs.
four minimal solvers [33, 47, 89]: P3P, P2P1LL, P1P2LL, P3LL from PoseLib [35], together in a hybrid RANSAC framework [10,59] with local optimization $[11,36]$ to get the final 6-DoF pose (refer to Sec. H in supp. for details). This also enables direct comparison since only using P3P [47] corresponds to the point-alone baseline similar to $\operatorname{HLoc}[56,57]$. We also compare with the post-refinement of PtLine [17] that optimizes over the initial point-alone predictions.

Results in Tab. 7 show that our localization system achieves consistently better results than the point-alone baseline both indoors [66] and outdoors [30], validating the effectiveness of employing 3D line maps for visual localization. In Fig. 9 we show more detailed results from the Stairs scene from 7Scenes [66] as it is one of the most challenging ones. Integrating lines significantly benefits the alignment of the reprojected structures, improving the pose accuracy from 46.8 to 71.1. Also, with our localization pipeline, using the map built from our proposed method is better than from L3D++ [23] by a noticeable margin, again demonstrating the advantages of our proposed line mapping system.
Refining Structure-from-Motion. With the acquired 3D line maps built from a roughly correct point-based structure-from-motion model, e.g, COLMAP [64], we can use the 3D lines with their track information to refine the input camera poses with joint optimization of points and lines. To verify this, we run COLMAP [64] with SuperPoint [13] on the first eight scenes of Hypersim [55], run the proposed line mapping on top of it, and perform joint bundle adjustment to refine poses and intrinsics. We report the relative pose evaluation of all image pairs [29]. Tab. 8 shows that the joint point-line refinement consistently benefits the accuracy of the camera poses, in particular improving AUC@ $1^{\circ}$ by 5.6.

## 5. Conclusion

In this paper, we introduce LIMAP: a library for robust 3D line mapping from multi-view imagery. Extensive experiments show that our method, by improving all stages of the reconstruction pipeline, produces significantly more complete 3D lines, with much higher quality of track association. As a byproduct, the method can also recover 3D association graphs between lines and points / VPs. We further show the usefulness of 3D line maps on visual localization and bundle adjustment. Future directions include incremental / real-time structure mapping, distinguishing structural lines from textural lines for wireframe modeling, and exploiting higher-level structures and relations for downstream applications.
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[^0]:    ${ }^{1}$ https://skyearth.org/publication/project/ELSR/

[^1]:    ${ }^{2} \mathrm{Up}$ to the date of submission, the COLMAP model [64] used by HLoc [56,57] does not consider radial distortion from the VisualSfM [80] model. So our results are better than the original ones.

